is extensively dealt with in Chapter 3. Important sections here are devoted to the case of nearby singularities, to the phenomenon of avoided crossings in eigenvalue problems for the triconfluent Heun equation, and to the two-parametric eigenvalue problem arising with the doubly confluent Heun equation. Chapter 4 presents a selection of (eigenvalue) problems in physics, astrophysics, and celestial mechanics where confluent versions of Heun's equation show up. Chapter 5 is devoted to the one-to-one correspondence between Heun-class equations (interpreted as Hamiltonians) and equations from the Painlevé class (interpreted as equations of motion).

The book is concisely written, omitting long proofs and reducing critical ones to the essentials. With its numerous tables, lists, and schemes it can be considered as a useful reference work for researchers in applied mathematics and physics. The software package SFTools, announced in Appendix D, might be of special interest to them.

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doi:10.1006/jath.2001.3605

Yves Nievergelt, *Wavelets Made Easy*, Birkhäuser, Boston, 1999 (2nd printing with corrections 2001), xi+297 pp.

During the past few years several books on wavelets have appeared. The book under review deserves special attention because it really lives up to what its title promises: it makes wavelets easy. The author claims that the material presented addresses the audience of engineers, financiers, scientists, and students looking for explanations of wavelets at the undergraduate level. The more advanced researcher, and in particular approximation theorists, will need supplementary material, but will nevertheless like the lucid presentation.

The book consists of three parts, A, B, and C (and a mysterious part D called *Directories* which consists of an empty page). Part A (Chapters 1–3) deals with *Algorithms for Wavelet Transforms*. The author starts with a chapter devoted to Haar wavelets, which are worked out in detail. Very little calculus or linear algebra is needed to explain these Haar wavelets, but they allow a clear explanation of the general nature of wavelets and the fast wavelet transform, illustrated by examples from creek water temperature analysis and financial stock index event detection. In Chapter 2 the simple Haar wavelet is extended to two-dimensional Haar wavelets, with the help of some linear algebra. Applications in noise reduction, data compression, and edge detection are explained, with some computational notes, and some extra examples on two and three dimensional diffusion analysis are given. Chapter 3 deals with Daubechies wavelets (in one or more dimensions), with emphasis on the calculation (the fast Daubechies wavelet transform). This chapter also shows the need for some theory for the clarification of Daubechies wavelets, and this will be done in parts B and C.

Part B (Chapters 4–6) is about *Basic Fourier Analysis* and deals with the classical theory of least squares approximation with trigonometric functions. In fact, wavelets hardly appear, but it is clear that the results and techniques of this part will be used in part C for the computation and the design of wavelets. Chapter 4 gives the basic ingredients of Fourier analysis, such as inner product spaces, Gram–Schmidt orthogonalization, and orthogonal projections, and it is shown how orthogonal projections are used in three-dimensional computer graphics, in least-squares regression, in the computation of functions, and (of course) for building wavelets. Chapter 5 describes the discrete Fourier transform and its computation using the fast Fourier transform technique of Cooley and Tukey. The theory of Fourier series for periodic functions is worked out in Chapter 6, with some notions of convergence and inversion of Fourier series (such as the Gibbs–Wilbraham phenomenon).

As mentioned above, part C (Chapters 7–9) is about the *Computation and Design of Wavelets* and shows how the Fourier analysis of part B is used in wavelet design. Chapter 7 presents the Fourier transform and its inversion on the line and in space. In Chapter 8 it is explained how Daubechies used this Fourier transform to construct wavelets. The existence, uniqueness, and orthogonality of Daubechies wavelets is proved. It is also shown how other wavelets can be designed using the Fourier transform. Finally, Chapter 9 shows that wavelets can approximate signals accurately.

The author has indeed succeeded in writing a book for the intended audience. This is highly recommended as a textbook for an undergraduate course on (Fourier analysis and) wavelets.

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doi:10.1006/jath.2001.3606

W. Freeden, T. Gervens, and M. Schreiner, *Constructive Approximation on the Sphere, with Application to Geomathematics*, Numerical Mathematics and Scientific Computation, Oxford University Press, 1998, xv + 227 pp.

Geomathematics is a rather recent discipline which deals with mathematics concerned with scientific problems from geochemistry, geodesy, geology, and geophysics. This books give a comprehensive treatment of spherical approximation in (geo)mathematics with emphasis on the theory of spherical harmonics and approximation by splines and wavelets. In the preface the authors say:

> *This book provides the necessary foundation for students interested in any of the diverse areas of constructive approximation in sphere-oriented geomathematics. It is designed as a graduate-level textbook and assumes some basic undergraduate training in linear algebra and (functional) analysis, plus some basic knowledge of numerical analysis. But the primary objective of the book is to help geophysicists and geo-engineers to understand future aspects of approximation by spherical harmonics and their modern counterparts (such as splines, wavelets, etc.).*

To achieve this goal, the textbook is divided into three parts, which are quite different in size. Part I, on *scalars*, of about 300 pages, contains 10 chapters; part II, on *vectors*, contains 34 pages and two chapters; part III, on *tensors*, has 45 pages and two chapters (4 pages for the last chapter). This difference in size can be explained by the observation that much of the theory of spherical harmonics, splines, and wavelets has a straightforward extension to vectors and tensors. Extensions of scalar results to the vectorial or tensorial case are discussed in more detail only if they are not straightforward. There is also a somewhat less distinctive division into a part dealing with the theory of spherical harmonics (Chapters 1–4 in the scalar case, Chapter 12 for vectors, and Chapter 14 for tensors), spherical spline theory (Chapters 5 and 6 for scalars, Chapter 13 for vectors, Chapter 15 for tensors), and spherical wavelet theory (Chapters 5, 7–11 for vectors, Chapter 13 for vectors, and Chapter 15 for tensors).

Most of the important properties of spherical harmonics can be described using homogeneous harmonic polynomials (Chapter 2). Spherical geometry is closely related to the orthogonal group $O(3)$ of real 3×3 orthogonal matrices and the special orthogonal group $SO(3)$ of orthogonal matrices with determinant 1. This approach is described in Chapter 3, where it is shown that spherical harmonics of given order form an irreducible invariant subspace of the space of square-integrable functions on the unit sphere. Spherical harmonics as eigenfunctions of the Beltrami operator, integral formulas, and the theory of Green's functions on the unit sphere are considered in Chapter 4.

Chapter 5 deals with spherical basis functions, with a development of general Sobolev spaces and invariant pseudodifferential operators which allows to recognize the radial basis function as an axisymmetric reproducing kernel function of the Sobolev space, which admits a Legendre expansion determined by the *spherical symbol* of the pseudodifferential operator. This chapter contains several examples of radial basis functions. Splines for spherical